RCA

Club Rates may be had on Application

Mathematics News Letter

SUBSCRIPTION $1.00 PER YEAR Single Copies..15c.

Published eight times per year at Baton Rouge, Louisiana.

Dedicated to mathematics in general and to the following aims in particular: (1) a study of the common problems of secondary and collegiate mathematics teaching, (2) a true valuation of the disciplines of mathematics, (3) the publication of high class expository papers on mathematics, (4) the development of greater public interest in mathematics by the publication of authoritative papers treating its cultural, humanistic and historical phases.

EDITORIAL BOARD

S. T. SANDERS, Editor and Manager, L. S. U., Baton Rouge, La.
H. E. BUCHANAN, Tulane University, New Orleans, La.
T. A. BICKERSTAFF, University, Miss.
W. VANN PARKER, Miss. Woman's College, Hattiesburg, Miss.
W. E. BYRNE, Virginia Military Institute, Lexington, Va.
RAYMOND GARVER, University of California at Los Angeles, Los Angeles, Cal.
C. D. SMITH, A. & M. College, Miss.
F. A. RICKEY, Mandeville, La.
IRBY C. NICHOLS, L. S. U., Baton Rouge, La.
H. LYLE SMITH, L. S. U., Baton Rouge, La.
W. PAUL WEBBER, L. S. U., Baton Rouge, La.
WILSON L. MISER, Vanderbilt University, Nashville, Tenn.
MRS. MARTIN L. RILEY Baton Rouge, La.
J. McGIFFERT, Rensselaer Polytechnic Institute, Troy, N. Y.

VOL. 7 BATON ROUGE, LA., NOVEMBER, 1932 No. 2

CONTENTS

The Data-Examining Habit

Mathematicians and Their Inspirations

The Louisiana-Mississippi Section of the M. A. of A. and the Next Program

Teaching Subtraction of the Signed Numbers

Teaching Mathematics for Discipline

Humanizing Mathematics

On Lines of a Triangle

The Parametric Representation of a Certain Parabola

The Dialytic Method for Solving Simultaneous Equations

The Approximation of Real Roots of Equations by Simple Fractions

Problem Department

THE DATA-EXAMINING HABIT

Again and again have we seen the flash of joyous insight take the place of visible perplexity on the student's face when he has been forced to confront squarely the facts implied by the hypothesis of his problem. Almost unlimited is the number of problems in elementary mathematics whose solutions are so little removed from the premises that only a detailed statement of them is sufficient to start the needed train of inference that leads to the solution.

Are there other fields in which the habit may function with valuable effect? Whatever may be the setting or the content of a situation that offers a PROBLEM ELEMENT to a mind trained in problem-solving, that mind must be led by its data-examining habit to take no forward step until all available means for attaining the objective have been surveyed. Such a mind will subject to orderly array all contingent factors entering into the situation. To secure the fullest expression to the influence of factors which are not contingent, but SURE in their operations, these also will be surveyed and classified, and their relative effects will be carefully noted as far as it can be done. The mind so trained, on undertaking a complicated business enterprise, will not move forward until complete inventory has been made of legitimate agencies for securing a success of the enterprise. As far as possible, the certain influence of each agency will be measured with accuracy, so that a corresponding accurate measure of its bearing on the desired success may be had. Contingent factors will be reduced, and contingency, itself, carefully weighed. If a mind so trained should be called to lead a military enterprise, the data-examining habit would assert itself. Tactics, of both the contingent and the certain sort, would be enumerated, examined, classified, and their relative efficiencies measured by the degree of their applicability to the military objective. Actual situations without limit in every department of life could be cited which would appeal to a mind thus trained.—S. T. S.

MATHEMATICIANS AND THEIR INSPIRATIONS

Tartaglia was inspired to a solution of the general cubic equation by his preparation for a contest in mathematical problem solving. The challenge came from Antonio del Florido, who knew only an empirical solution of $x^3+qx=r$. In the contest Tartaglia solved all the problems, Florido solved none.

Ferrari—famed for his solution of the fourth degree general equation—found his ability in mathematics by listening to Cardan's lectures while serving him as an errand boy.

Vieta—just as in Tartaglia's case—was inspired to some of his best work by a problem-solving challenge.

Again it was Descarte's success in responding to a challenge offered to the world to solve a certain geometrical problem that decided him to abandon the army and devote his life to mathematics.

Simpson, the English mathematician, had his first interest in mathematics aroused by a study of the eclipse of the sun in 1724.

Lagrange's serious attention to mathematics was occasioned by a reading of the astronomer Halley's memoir "On the Excellence of the Modern Algebra in Certain Optical Problems". Inspired by this tract Lagrange studied mathematics so diligently that at the end of a year, according to the historian Ball, he was "an accomplished mathematician".

At the age of 13, William Rowan Hamilton discovered by accident a copy of Newton's Universal Arithmetic. It was the reading of this that inspired him to study analysis. He became the founder of the theory of quaternions.

According to David Brewster, Isaac Newton did not begin seriously to study at school until one day when he suffered a painful blow from a school-fellow who ranked above him. Newton's retaliation took the form of an intense desire to outrank his enemy. This he did in time, by hard study.

Pascal's initial interest in mathematics was especially whetted by a summary order from his father to have nothing to do with mathematical books. To enforce his order the father kept such books concealed from the son. One day young Pascal asked his parent to tell him what mathematics was about. The answer was that "it was the method of making

figures with exactness, and of finding out what proportions they relatively had to one another". This answer inspired the boy to ponder in secret on the nature and properties of various geometric figures. At the age of 16 he wrote a masterly treatise on the conics.

It was a challenge issued by the French Academy to the mathematicians of the world to solve a certain astronomical problem that led Euler to solve it in three days' time. The intensity of his study caused the loss of his right eye—a deprivation that seemed only to stimulate him to greater mathematical activity.—S. T. S.

THE LOUISIANA-MISSISSIPPI SECTION OF THE M. A. OF A. AND THE NEXT PROGRAM

By C. D. SMITH
Louisiana-Mississippi Section

A Section of mathematics teachers must always depend for existence upon the program of the colleges in the territory. For example the financial program of colleges could be so reduced that the financial status of teachers would nullify active support of a Section. The business of any group is to defend the causes of education by presenting basic facts to the leaders who must function in a crisis. We shall not see another financial crisis so threatening the very foundations of college life as is now the case. Bankers are fighting as a group for their interests even to the point of securing government aid. Business is fighting for life even to the point of securing lower assessments and concessions relative to tax payments. Politicians are fighting for favor by proposing to sacrifice anything that would meet with public favor. The method is to propose a cut on everything and let the ax fall where the opposition seems to be the least active. In a crisis and under these conditions it is suicide for educators to wait for others to present needs and necessities.

What has this to do with the M. A. of A.? In public institutions budgets have been cut and defaulted almost to the point of starvation. The endowed institution has a skeleton of shrunken assets and patrons who have not paid fees. Good will has failed and the net result is the same as that of state programs. Who except the teacher will chamption the cause of leadership for tomorrow? In other words if teachers fail

today how shall they teach their students to succeed tomorrow? But, we say, the purpose of associations of mathematics teachers is to publish the results of their studies and promote progress in content and method. The idea seems to be that others will present the needs of the program. It is quite true that research in content and method is a laudable purpose. But I have observed another thing. Association with the Louisiana-Mississippi Section since its organization has revealed to me a desire on the part of members for cooperation of colleges and secondary schools in promoting the welfare of mathematical education. But survival must precede activity. The publication of the NEWS LETTER is one mode of attack on the problem. The annual meeting of the Section in joint session with the Council is another avenue of approach. Active support of the whole college program where standards are concerned is a basic principle. Our next meeting is at the Louisiana Polytechnic Institute, to be held next spring. Will you be there?

I am writing to suggest the necessity of cooperation on the part of every teacher in the two states. As a professional you are charged with a share in leadership and if you do not know just what to do then let us counsel together. The present condition constitutes a challenge to those who have given a major portion of time to research. A departure in our next program will be to provide a place for members to announce results of special study. One purpose of the present note is to ask all who wish to announce results to send a statement to Secretary Deborah May Hickey, Cleveland, Mississippi, prior to March first. Now in addition to this special activity I am suggesting that teachers owe something else to their students and to the schools which they are serving by way of adding to the program a plea for the future of the schools. We cannot perpetuate mathematics by failing to develop mathematicians. No mathematics is useful except as the mathematicians show how to use it. And so I ask for this year a renewed interest in the work of our Section, and that you come to Ruston next spring with a word and an idea. We well know that our government cannot perpetuate itself without fostering the higher institutions for development of leaders. We must realize that the institutions can be perpetuated only by leaders. Our duty is exceedingly plain. I am suggesting that we lend all possible support to executives and Boards of Control in their fight to save the colleges.

TEACHING SUBTRACTION OF SIGNED NUMBERS

By LENA R. COLE
Central Normal College
Danville, Indiana

I find, every year, that many students entering college algebra classes cannot subtract positive and negative numbers. They have a confused idea that "to subtract we add, with the sign of the subtrahend changed", but have no comprehension of why such a weird process *is* subtraction.

While it is true that this is just what all of us do, automatically, *after* we habituate the process, it certainly is no way to *explain* subtraction in the first place. Students in high school must be made to see *why* such an apparently strange thing happens to the signs.

Why not base our explanation entirely on the "additive" *meaning* of subtraction? Children in the first and second grades are taught that subtraction has *three* meanings:

(1) The "take away" meaning.

(2) The comparison meaning, as, 5 is how much greater than 3?

(3) What number *added* to 3 will give 5?

Since children have been familiar with this third meaning of subtraction since the first or second grade, why not make use of it in algebra, and discard *all* attempted illustrations, such as thermometers, balloons, graphs, debts, etc.? I believe that these concrete illustrations only confuse the pupil as to whether he is adding or subtracting. If we assume a man has ten dollars and owes three, we can very well illustrate addition of positive ten and negative three, but the situation really becomes absurd if we try to illustrate subtraction.

If, however, we ask the student what must be *added* to negative 3 to give positive ten, he answers at once, positive thirteen, and sees that the result of *subtracting* negative three from positive ten *must* be positive thirteen. He thus sees that there really is nothing new here, but merely subtraction such as he learned it in arithmetic.

TEACHING MATHEMATICS FOR DISCIPLINE

By W. PAUL WEBBER

Pursuant to the suggestion in the first paper on this topic I wish to cite further examples of the use of logical procedure in fields that are traditionally not considered as mathematical

One of Professor Moore's illustrations is very apt and is here reproduced. See Education, December, 1918: "In a speech in congress in 1828 Calhoun argues in effect as follows: The constitution is an agreement between sovereign states and by express provision of the constitution it cannot be changed without the consent of three-fourths of the states. Congress and other departments of government are creatures of the constitution and, adopted only to carry out its provisions, have no power to change it or to change the conditions of the parties thereto. Therefore in attempting to make such a change they would commit an act of usurpation. The main argument, stated in syllogistic form runs as follows: Whoever assumes a sovereign power to which he has no rightful claim is a usurper. Congress, in attempting to alter the constitution or the conditions of the parties thereto, would be assuming such power. Therefore, congress would be a usurper. This is entirely analogous to following argument in plane geometry: Any angle inscribed in a semi-circle is a right angle. This particular angle is inscribed in a semi-circle. Therefore this angle is a right angle."

Any one can see that the form of the above two syllogisms is the same, although they refer to entirely different subject matter. Professor Moore says "It is idle to assert that familiarity with the latter type of argument", the geometrical, "does not make it easier to understand the nature of the former one", the political one. To this I agree, but is this knowledge sufficient to enable or suggest to the average pupil to carry over the geometric argument to the political situation. For the average pupil I doubt it. In teaching geometry do we teach pupils how to analyse things ungeometrical and to reduce them to syllogistic form? I was not so taught and what little of that idea I got was due to my own effort later in life. If I, within two or three years after studying geometry, had been asked to analyse an argument such as Professor Moore's above, I should have been "stumped", and it would have taken me hours to do it if I did it at all.

While I agree with Professor Moore, I do not believe he goes far enough to make his theory (and it is the theory of the majority of teachers of mathematics) effective for the purpose which we have in mind, namely, the part of the fundamental principal of the National Committee that starts just following the word "environment". I doubt if a large percent of geometry students know what a syllogism is or what it signifies, to say nothing of being able to reduce the proofs of geometry to syllogistic form. By a knowledge of the syllogism I mean an intelligent idea of the syllogism as representing a fundamental thought process. You may say I was unfortunate in not having attended the right kind of schools. As to that I think the schools I attended were at least average in their pedagogical light for their day. Again I ask is it not rather strange that we have reached the stage of almost eliminating geometry from the high school curriculum without having sold our goods or making it evident even to the scientific minds generally that we are doing what we claim. Surely there must be a way of teaching every school subject so that pupils may realize some general human value before they grow gray.

To cite another example from the history of the race. Long ago certain philosophers taught that the earth and all the planets moved in circles. This is how they proved it: "Everything that God made is perfect. The circle is the only perfect curve. Therefore the planets must move only in circles." This argument was taken seriously by a lot of people. Suppose a class in geometry were given this argument to analyse. It might be as interesting and as profitable as collecting fifteen or twenty different proofs of the Pythagorean Theorem. The latter has often been done in geometry classes. In regard to the above argument it has been found by actual observation that none of the planets move in circles, but rather in ellipses. Wherein is the above argument weak? Would not examples of this general type be a valuable adjunct to the teaching of geometry?

"HUMANIZING MATHEMATICS"

Compiled by
PROFESSOR R. C. ARCHIBALD
Brown University

T. P. Sharwell, "Books that help make mathematics interesting", *Mathematics Teacher*, v. 19, Nov. 1926, p. 419-428.

F. S. Marvin, *The Living Past*, Fourth ed., Oxford, 1920. [Valuable for indicating the place of mathematics in the history of civilization.]

National Council of Teachers of Mathematics. The Third Yearbook, New York, 1928. [W. S. Schlauch, "Mathematics as an interpreter of life", p. 24-34; M. Gugle, "Dynamic Symmetry", p. 57-64.]

National Council of Teachers of Mathematics. The Sixth Yearbook. Mathematics in Modern Life, New York, 1931. [Contents: I. Fisher, "The application of mathematics to the social sciences"; J. A. Harris, "Mathematics in biology"; C. J. Keyser, "The humanistic bearings of mathematics"; D. E. Smith, "Mathematics and religion"; W. L. Hart, "The mathematics of investment"; H. B. Roe, "Mathematics in agriculture"; E. Spease, "Mathematics in pharmacy and in allied professions"; H. M. Walker, "Mathematics and statistics"; H. E. Brown, "Mathematics in physics"; G. D. Birkhoff, "Polygonal forms".]

E. T. Browne, "Human elements in mathematics", *Mathematics Teacher*, v. 24, Feb. 1931, p. 103-109.

C. C. Camp, "Contributions of mathematics to modern life", *Mathematics Teacher*, v. 21, April, 1928, p. 219-226.

A. J. Kemper, "The cultural value of mathematics", *Mathematics Teacher*, v. 22, March, 1929, p. 127-145.

C. N. Moore, "Mathematics and the future", *Mathematics Teacher*, v. 22, April, 1929, p. 203-214.

G. B. Olds, "Mathematics and modern life", *Mathematics Teacher*, v. 21, April 1928, p. 183-196.

W. W. Rankin, "The cultural value of mathematics", *Mathematics Teacher*, v. 22, April, 1929, p. 215-223.

H. E. Slaught, "Mathematics and sunshine", *Mathematics Teacher*, v. 21, May, 1928, p. 245-252.

H. E. Slaught, "Romance of mathematics", *Mathematics Teacher*, v. 20, Oct. 1927, p. 303-309.

N. B. Freeman, "Living mathematics", *Mathematics Teacher*, v. 23, Dec. 1930, p. 492-495.

L. B. Hansen, "Creating an interest in mathematics through special topics," *Mathematics Teacher*, v. 23, Jan. 1930, p. 3-6.

W. I. Schaaf, "Mathematics and world history", *Mathematics Teacher*, v. 23, Dec. 1930, p. 496-503.

A. S. Adams, "Civic values in the study of mathematics", *Mathematics Teacher*, v. 21, Jan. 1928, p. 37-41.

E. J. Martin, "Mathematics in industry and research", *Mathematics Teacher*, v. 22, Mar. 1929, p. 172-177.

E. Clarke, "Mathematics in modern business", *Mathematics Teacher*, v. 21, May 1928, p. 259-267.

W. W. R. Ball, *Mathematical Recreations and Essays*, Fifth ed., London, 1911.

M. E. Boole, *Philosophy and Fun of Algebra*, London, 1909.

H. E. Dudeney, *Amusements in Mathematics*, London, 1917.

H. E. Dudeney, *The Canterbury Puzzles and Other Curious Problems*, Second ed., London, 1919.

H. E. Dudeney, *Modern Puzzles and How to Solve Them*, London, 1926.

C. W. Newhall, "Recreations in secondary mathematics", *School Science and Mathematics*, v. 15, Dec. 1915, p. 277-293 (good bibliography).

M. Pierce, "Mathematical recreations", *Mathematics Teacher*, v. 20, Jan. 1926, p. 13-24.

T. Dantzig, *Number the Language of Science, A Critical Survey Written for the Cultured Non-Mathematician*, New York, 1930.

F. D. Mitchell, "Mathematical prodigies", *Amer. Jl. Psychology*, v. 18, Jan. 1907, p. 61-143.

M Sykes, *A Source Book of Problems for Geometry Based Upon Industrial Design and Architectural Ornament*, Boston, 1912.

D. E. Smith, *Mathematica Gothica*, Paris, 1925. 30 p.

L. D. Caskey, *Geometry of Greek Vases*, Boston, 1922.

W. R. Ransom, "Geometry measures land", *Mathematics Teacher*, v. 23, April 1930, p. 243-251.

Sister Alice Irene, "Some objectives to be realized in a course in plane geometry", *Mathematics Teacher*, v. 22, Dec. 1929, p. 435-446.

Erma Scott, "Geometry humanized, a school play in one act", *Mathematics Teacher*, v. 22, Feb. 1928, p. 92-101.

T. S. Row, *Geometrical Exercises in Paper Folding*, Second ed., Chicago, 1905.

A. Seybold, "The fourth dimension: An article for high school geometry teachers", *Mathematics Teacher*, v. 24, Jan. 1931, p. 41-45, (other references on page 45.)

E. A. Abbott, *Flatland: a Romance of Many Dimensions*, Third ed., rev., Oxford, 1926.

L. R. Lieber, *Non-Euclidean Geometry; or Three Moons in Mathesis*, New York, 1931.

J. M. Laird, "Mathematics as used by the actuary", *Mathematics Teacher*, v. 16, Dec. 1923, p. 500-503.

H. Cole, "Practical applications of mathematics to engineering problems", *Mathematics Teacher*, v. 25, May 1932, p. 290-297.

P. H. Nygaard, "Radio and airplane distances", *Mathematics Teacher*, v. 22, Feb. 1929, p. 117-120.

D'A. W. Thompson, *On Growth and Form*, Cambridge, 1917.

T. A. Cook, *The Curves of Life Being An Account of Spiral Formations and their Application to Growth in Nature to Science and to Art*, London, 1914.

Sister Alice Irene, "Mystic mathematics, the message of the spiral", *Mathematics Teacher*, v. 24, Mar. 1931, p. 139-150.

D. E. Smith, "Esthetics and mathematics", *Mathematics Teacher*, v. 20, Dec. 1927, p. 419-428.

D. E. Smith, "Religio mathematici", *Amer. Math. Monthly*, v. 28, Oct. 1921, p. 339-349; and Columbia Univ., *Teachers College Record*, Nov. 1921.

J. W. N. Sullivan, *Aspects of Science* (a) First series, London, 1927, ("Mathematics and music", p. 185-205); (b) Second series, New York, 1926 ("Mathematics as an art", p. 80-96.)

S. Buchanan, *Poetry and Mathematics*, New York, 1929, p. 3-24.

D. E. Smith, "Poetry of Mathematics", *Mathematics Teacher*, v. 19, May 1926, p. 291-296.

ON LINES OF A TRIANGLE

By W. V. PARKER
Mississippi Woman's College

The following problem was proposed for solution in the issue of the *Monthly* for January, 1931.

"If the hypotenuse of a right triangle be divided into n equal parts and the vertex of the right angle joined to these points of equal division, then, if d_i be the length of the lines so drawn

$$\sum_{i=1}^{n-1} d_i^2 = \frac{(n-1)(2n-1)}{6n} h^2$$

where h is the length of the hypotenuse."

In the solution of this problem attention was called to a more general theorem of which the above is a special case. This theorem may be stated as follows:

If each of the three sides of a triangle ABC be divided into n equal parts and the oppostire vertices joined to these points of equal division, then, if a_i, b_i, c_i be the lengths of the lines so drawn from A, B and C respectively

$$\sum_{i=1}^{n-1} a_i^2 = (n-1)\,bc\,\mathrm{Cos}\,A + \frac{(n-1)(2n-1)}{6n} a^2,$$

$$\sum_{i=1}^{n-1} b_i^2 = (n-1)\,ac\,\mathrm{Cos}\,B + \frac{(n-1)(2n-1)}{6n} b^2,$$

$$\sum_{i=1}^{n-1} c_i^2 = (n-1)\,ab\,\mathrm{Cos}\,C + \frac{(n-1)(2n-1)}{6n} c^2,$$

where a, b, c are the lengths of the sides of the triangle.

Combining these relations and making use of the fact that

$$(n-1)\ bc\ \text{Cos}\ A = \frac{n-1}{2}(b^2+c^2-a^2),$$

$$(n-1)\ ac\ \text{Cos}\ B = \frac{n-1}{2}(a^2+c^2-b^2),$$

$$(n-1)\ ab\ \text{Cos}\ C = \frac{n-1}{2}(a^2+b^2-c^2),$$

we get at once

$$\sum_{i=1}^{n-1}(a_i^2+b_i^2+c_i^2) = \frac{(n-1)(5n-1)}{6n}(a^2+b^2+c^2).$$

In particular, for $n=2$, we have the well known theorem that the sum of the squares of the medians of a triangle is equal to three-fourths the sum of the squares of the sides.

If the subscripts be taken consecutively from 1 to $n-1$ in a counter clockwise direction about each of the vertices and the intersection of a_i with b_{n-i} denoted by C_i, the intersection of b_i with c_{n-i} denoted by A_i, and the interesction of c_i with a_{n-i} denoted by B_i, triangles ABC and $A_iB_iC_i$ are homothetic (that is similar with corresponding sides parallel) and the median point of ABC is the homothetic center or center similitude.

THE PARAMETRIC REPRESENTATION OF A CERTAIN PARABOLA

By W. E. BYRNE

In many elementary texts on the calculus we find one or more problems of deducing the relation between x and y which is equivalent to a given parametric representation of a curve. In particular the

following problem appears frequently to eliminate: t between the equations

(1) $\quad x=(t-a)^2,\ y=(t-b)^2,\ -\infty<t<t\infty$

For the sake of argument we shall suppose that $b>a$, and we shall use the symbol $\sqrt{\ }$ to mean the principal determination (positive or zero). Generally the answer indicated in texts is incorrect.

We may replace the given equations by the following:

(2) $\quad t-a=\sqrt{x}$ $\quad$ where $t \lessdot a$

(3) $\quad t-a=-\sqrt{x}$ $\quad t \gtrdot a$

(3) $\quad t-a=-\sqrt{x}$ $\quad t \gtrdot a$

(4) $\quad t-b=\sqrt{y}$ $\quad t \lessdot b$

(5) $\quad t-b=-\sqrt{y}$ $\quad t \gtrdot b$

This gives us three different cases, to-wit:

Case I. $\quad t \gtrdot a$

$$t=a-\sqrt{x}=b-\sqrt{y}$$

(6) $\quad \sqrt{y}-\sqrt{x}=b-a$

Case II. $\quad a \gtrdot t \gtrdot b$

$$t=a+\sqrt{x}=b-\sqrt{y}$$

(7) $\quad \sqrt{x}+\sqrt{y}=b-a$

Case III. $\quad t \lessdot b$

$$t=a+\sqrt{x}=b+\sqrt{y}$$

(8) $\quad \sqrt{x}-\sqrt{y}=b-a$

The curve may be found directly from the parametric euqations as indicated below:

t	$-\infty$		a		b		$+\infty$
$\frac{dx}{dt}$	$-$		0		$+$		$+$
x	$+\infty$	$\searrow$	0	$\nearrow$	$(b-a)^2$	$\nearrow$	$+\infty$
$\frac{dy}{dt}$	$-$		$-$		0		$+$
y	$+\infty$	$\searrow$	$(a-b)^2$	$\searrow$	0	$\nearrow$	$+\infty$

Case I corresponds to the infinite branch CA, Case II to the arc AB, and Case III to the infinite branch BD, shown in Figure 1.

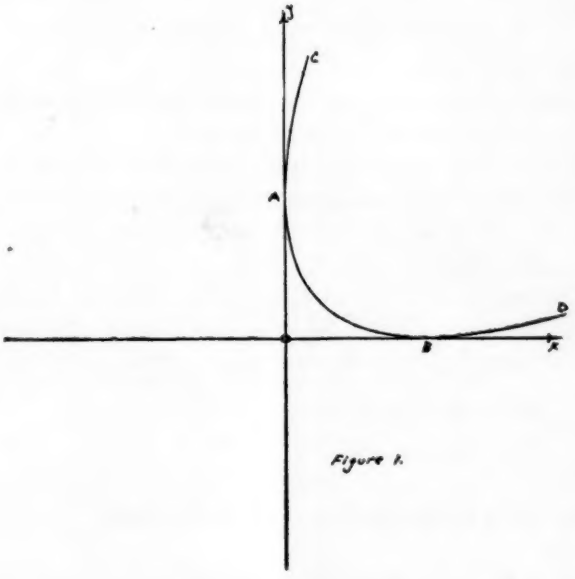

Figure 1.

If we wish to check the concavity a short calculation gives

$$\frac{d^2y}{dx^2} = \frac{b-a}{2(t-a)^3} \qquad \text{provided } t \neq a \tag{9}$$

and

$$\frac{d^2x}{dy^2} = \frac{a-b}{2(t-b)^3} \qquad \text{provided } t \neq b \tag{10}$$

From the point of view of anyone who wishes to apply his mathematics to get consistent results in geometry or physics it is regrettable that "run of mine" college algebras, trigonometries, analytic geometries and calculus texts do not apply principal determinations of square roots outside of some of the derivations.

THE DIALYTIC METHOD FOR SOLVING SIMULTANEOUS EQUATIONS

By WILSON L. MISER
Vanderbilt University

The following theorem may be made the basis for solving simultaneous equations in two or more unknowns.

Theorem: The necessary and sufficient condition that n linear homogeneous equations in n unknowns shall have a solution, other than the trivial one in which each unknown is zero, is that the determinant of the coefficients be zero.

This theorem is proved in such text books as Bocher's Higher Algebra and Dickson's First Course in the Theory of Equations.

The following examples illustrate the common method:

Example 1. Solve $3x+4y=10$,

$$4x+y=9.$$

To solve for x write the equations in the form

$$(3x-10)1+4y=0,$$

$$(4x-\ 9)\ +y=0,$$

which is homogeneous in 1 and y. In order that 1 and y may not both be zero, we must have

$$\begin{vmatrix} 3x-10 & 4 \\ 4x-9 & 1 \end{vmatrix} = 0.$$

This equation in x has the solution $x=2$.

To solve for y write the equations in the form

$$3x+(4y-10)1=0,$$
$$4x+(y-9)1=0.$$

In order that x and 1 may not both be zero, we must have

$$\begin{vmatrix} 3 & 4y-10 \\ 4 & y-9 \end{vmatrix} = 0.$$

The solution of this equation is $y=1$.

Hence the given system has the solution $x=2$, $y=1$.

Example 2. Solve $x-2y=2$,

$$xy=4.$$

To solve for x write the equations as homogeneous in 1 and y.

$$(x-2)1-2y=0,$$
$$(-4)1+xy=0.$$

Setting the determinant equal to zero, we have

$$\begin{vmatrix} x-2 & -2 \\ -4 & x \end{vmatrix} = 0,$$

which reduces to the quadratic equation

$$x^2-2x-8=0.$$

The roots are $x=4$, $x=-2$.

To solve the equations for y, write the equations as homogeneous in x and 1.

$$x+(-2y-2)1=0,$$
$$yx+(-4)1=0.$$

Setting the determinant equal to zero, the roots of the quadratic are found to be $y=1$, $y=-2$.

Pairing the values by the equation $xy=4$, the solutions of the given system are $x=4$, $y=1$; $x=-2$, $y=-2$.

Example 3. Solve $3x+y+4z=17$,
$$x+3y+3z=16,$$
$$x+2y+2z=11.$$

To eliminate x, write the equations as homogeneous in x and 1.

$$3x+(y+4z-171=0,$$
$$x+(3y+3z-16)1=0.$$
$$x+(2y+2z-11)1=0.$$

From the first and second equations and from the second and third, we have

$$\begin{vmatrix} 3 & y+4x-17 \\ 1 & 3y+3z-16 \end{vmatrix}=0, \qquad \begin{vmatrix} 1 & 3y+3z-16 \\ 1 & 2y+2z-11 \end{vmatrix}=0.$$

These equations reduce to

$$8y+5z-31=0,$$
$$y+z-5=0,$$

which may be solved for y and z.

The solution of the original system is $x=1$, $y=2$, $z=3$.

Example 4. Solve $4x^2+9y^2=36$,

$$2x-3y=6.$$

The system can be written as three homogeneous equations in x^2, x, and 1 by multiplying the second equation by x to obtain a new equation.

$$4x^2+0x+(9y^2-36)1=0,$$

$$2x^2+(-3y-6)x=0,$$

$$2x+(-3y-6)1=0.$$

In order that x^2, x, and 1 shall not all be zero, the determinant of the coefficients must be equal to zero.

$$\begin{vmatrix} 4 & 0 & 9y^2-36 \\ 2 & -3y-6 & 0 \\ 0 & 2 & -3y-6 \end{vmatrix} = 0.$$

This equation reduces to

$$y^2+2y=0,$$

whose roots are $y=0$ and $y=-2$.

Finding the values of x from the linear equation the solutions are $x=3$, $y=0$; $x=0$, $y=-2$.

Example 5. Solve $x^2+xy+2y^2=11$,

$$2x^2+5y^2=22.$$

A system of four simultaneous homegeneous equations in x^3, x^2, x, and 1 can be written by multiplying both equations by y to obtain two additional equations.

$$x^3+yx^2+(2y^2-11)x=0,$$

$$x^2+yx+(2y^2-11)1=0,$$

$$2x^3+0x^2+(5y^2-22)x=0,$$

$$2x^2+0x+(5y^2-22)1=0.$$

The determinant of fourth order set equal to zero reduces to the quartic equation $y^4-4y^2=0$. The values $y=0$, $y=2$, $y=-2$ give rise to six values for x of which two must be discarded in the pairing. The solutions of the given system are $x=\sqrt{11}$, $y=0$; $x=-\sqrt{11}$, $y=0$; $x=1$, $y=2$; $x=-1$, $y=-2$.

The common method here illustrated for solving simultaneous equations in two or more unknowns was suggested to the author by the exposition of Sylvester's Dialytic Method of Elimination as given by Dickson in the text book cited at the beginning of the paper. We should therefore call the method of this paper the Dialytic Method.

THE APPROXIMATION OF REAL ROOTS OF EQUATIONS BY SIMPLE CONTINUED FRACTIONS

By RAYMOND GARVER

One of the less important applications of simple continued fractions is to the approximation of real roots of equations. The method is due to Lagrange, and a discussion of it may be found in Weber's Algebra, vol. I, 1895, pp. 401-3. It is not mentioned, however, by most books on Higher Albegra and the Theory of Equations. Some sources, including the Encyclopedia Britannica, go so far as to say that the method is of no practical value.

This criticism, I feel, is too severe to be justified. Admittedly, the method often does not work out very satisfactorily, but in many other cases it gives a close approximation with a minimum of labor. Further, it allows us to estimate the accuracy of our approximation within two easily obtainable limits. Even in an upper class course in the Theory of Equations students often fail to realize the necessity of having some control over the accuracy of numerical work, and anything that emphasizes this is of some value.

As a first example, let us approximate the real root of $x^3-x-9=0$; this example, incidentally, is worked out in Cajori's Theory of Equations, pages 63-4. First, apply the transformation $x=2+1/x_1$ the root obviously lying between 2 and 3. Since $x_1=1/(x-2)$, we first

reduce the roots by two, and then apply the reciprocal transformation by simply reading the coefficient off backwards. Thus

$$\begin{array}{rrrrrr|l} 1 & +0 & -1 & -9 & & 2 \\ & 2 & 4 & 6 & & \\ \hline 1 & 2 & 3 & & -3 & \\ & 2 & 8 & & & \\ \hline 1 & 4 & 11 & & & \\ 1 & 2 & & & & \\ \hline 1 & 6 & & & & \end{array}$$

and we have $3x_1^3-11x_1^2-6x_1-1=0$. Now, by its very formation, the value of x_1 which we are seeking must be greater than 1; we see at once that it is between 4 and 5. Next set $x_1=4+1/x_2$, and the same process which we have just used leads to $9x_2^3-50x_2^2-25x_2-9=0$. Since x_2 is between 6 and 7, set $x_2=6+1/x_3$, which leads to $9x_3^3-347x_3^2-112x_3-9=0$. Now x_3 is large, in fact, between 38 and 39, and we do not have to carry through the next calculation, that is, the determination of x_4 from the relation $x_3=38+1/x_4$, to see that x_4 is between 1 and 2.

We now have the real root of the given equation in the form of a continued fraction.

$$x=2+\cfrac{1}{4+\cfrac{1}{6+\cfrac{1}{38+\cfrac{1}{1+\cdots}}}}$$

The fourth convergent gives the approximation $2137/954=2.2400417+$. Now it is well known, from the theory of continued fractions, that if

$$x=a_1+\cfrac{1}{a_2+\cfrac{1}{a_3+\cdots+\cfrac{1}{a_n+\cdots}}}$$

the error committed in taking the nth convergent, p_n/q_n, as the value of x is less than $1/q_nq_{n+1}$ and greater than $1/2q_nq_{n+1}$ in absolute value. Further, if n is odd, the nth convergent is smaller than x, while if n is even, it is larger.

Thus, seeing that $q_5 = a_5\, q_4 + q_3 = 954 + 25 = 979$, we can say at once that our approximation 2.2400417 is too large by an amount not greater than 1/954.979, or substantially .000001, and by an amount not less than half of this. Applying these as corrections to the fourth convergent, we can say definitely that the root is 2.240041, correct to six decimal places. We have had to make only three transformations, of the type used in Horner's method, but with the coefficients read in reverse order, and a part of the fourth transformation. We have not used decimals, as does Horner's method; the reader may be interested in working the problem by that method, or he may find the details in Cajori. On the other hand it must be pointed out that the continued fraction method will not be very convenient unless we find a large number, or several fairly large numbers, among the partial quotients $a_2, a_3, a_4, \ldots$ The reason for this is obvious.

For a second example, let us consider the approximation of the real cube root of 28, that is the determination of the real root of the equation $x^3 - 28 = 0$. The reader will have no trouble in showing, by the same process that was used in the preceding example, that the root is given by the continued fraction

$$x = 3 + \cfrac{1}{27 + \cfrac{1}{3 + \cfrac{1}{40 + \cfrac{1}{1 + \ldots}}}}$$

The fourth convergent is $10042/3307 = 3.03658905+$. This time $q_5 = a_5 q_4 + q_3 = 3307 + 82 = 3389$, and $1/q_4 q_5 = 1/3307.3389$, or substantially .00000009, as can be seen practically without computation. The fourth convergent is then too large by an amount not greater than .00000009, and by an amount at least half this great. Consequently, the real cube root of 28 is equal to 3.0365890, correct to seven decimal places.

In this example, the reader may interested in computing the cube root of 28 by the binomial theorem, and comparing that method with the preceding.

PROBLEM DEPARTMENT

Edited by
T. A. BICKERSTAFF
University, Miss.

This department aims to provide problems of varying degrees of difficulty which will interest anyone who is engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and solve problems here proposed.

Problems, and solutions will be credited to their authors.

Send all communications about problems to T. A. Bickerstaff, University, Mississippi.

Problems for Solution

No. 23. Proposed by William E. Byrne, Virginia Military Institute, Lexington, Va.

Find the most general solution of the simultaneous equations,

$$\cos\Phi = \frac{1+\cos\alpha}{2\left|\cos\frac{\alpha}{2}\right|}$$

$$\sin\Phi = \frac{\sin\alpha}{2\left|\cos\frac{\alpha}{2}\right|}$$

where $|\ |$ indicates absolute value and α is a constant, $\cos\frac{\alpha}{2} \neq 0$.

No. 24. Proposed by William E. Byrne, Virginia Military Institute, Lexington, Va.

Prove that if k is a given positive integer, the inequalities

$$\frac{(s-1)(s-2)}{2} < k > \frac{s(s-1)}{2}$$

admit one and only one positive integral solution s. Find this solution for $k = 1000$.

No. 25. Proposed by E. C. Kennedy, College of Mines, El Paso, Tex.

Solve for least positive value of x to within an error of about one second or less. The solution may be obtained very easily if the expression is simplified properly.

$$\sqrt{\sin 2x} = \sqrt{2}\cos x[\sqrt[u]{4} \cdot 76 - 1]$$

Where $u = 1 + \sqrt{\tan x}$